Photon-neutrino interaction in θ-exact covariant noncommutative model: Phenomenology and Quantum properties*

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We construct the θ-exact covariant noncommutative (NC) model and obtain various closed constraints on the NC scale (Λ_{NC}) from inelastic neutrino-nucleon scatterings ($\nu N \to \nu + X$), from plasmon decay into neutrino pair ($\gamma_{pl} \to \bar{\nu}\nu$), from the BBN nucleosynthesis and from the reheating phase after inflation, respectively. We have have found neutrino two-point function in a closed form. It shows decoupling of the UV divergent term from softened UV/IR mixing term and from the finite terms as well. We show that problematic UV divergent and UV/IR mixing terms vanish for a certain choice of the noncommutative parameter θ which preserves unitarity. We find non-perturbative modifications of the neutrino dispersion relations which are assymptotically independent of the scale of noncommutativity in both the low and high energy limits and may allow superluminal propagation.

Introduction

The study of spacetime quantization has originally been motivated by major problems of physics at extremely-high energies, in particular the problems of renormalization and quantum gravity. Heisenberg-type spacetime uncertainty relations can effectively lead to a replacement of the continuum of points by finite size spacetime cells, thus providing a means by which to tame UV divergences. It is reasonable to expect that noncommutative (NC) field theory models can provide some guidance for a deeper understanding of the structure of spacetime at extremely-high energies. In fact, these NC models appear quite naturally in string theory [1]. The relevant NC scale may very well be beyond direct experimental reach for the foreseeable future (except in certain theories with large extra dimensions). Nevertheless, non-perturbative effects can nevertheless lead to profound observable consequences for low energy physics. A famous example is UV/IR mixing [2, 3]. Another striking example is the running of the coupling constant of NC QED [4], rendering that theory asymptotically free. In this article we derive similarly non-perturbative result for ν-propagation in the NC spacetimes, including in particular modified dispersion relations that allow superluminal velocities as they are currently discussed in the context of the OPERA experiment [5].

In a simple model of NC spacetime local coordinates x^μ are promoted to hermitian operators $\hat{x}^\mu$ satisfying spacetime NC and implying uncertainty relations

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} \longrightarrow \Delta x^\mu \Delta x^\nu \geq \frac{1}{2}|\theta^{\mu\nu}|, \qquad (1)$$

where $\theta^{\mu\nu}$ is real, antisymmetric matrix.

The Moyal-Weyl $\star$-product, relevant for the case of a constant $\theta^{\mu\nu}$, is defined as follows:

$$(f \star g)(x) = f(x)e^{\frac{i}{2}\overleftarrow{\partial_\mu}\theta^{\mu\nu}\overrightarrow{\partial_\nu}}g(x). \qquad (2)$$

The operator commutation relation (1) is then realized by the $\star$-commutator

$$[\hat{x}^\mu, \hat{x}^\nu] = [x^\mu \stackrel{\star}{,} x^\nu] = i\theta^{\mu\nu}. \qquad (3)$$

In our construction the noncommutative fields are obtained via so-called Seiberg-Witten (SW) maps [1] from the original commutative fields. It is commutative instead of the noncommutative gauge symmetry that is preserved as the fundamental symmetry of the theory.

The perturbative quantization of noncommutative field theories was first proposed in a pioneering paper by Filk [6]. However, despite of some significant progress like the models in [7] and [8], a better understanding of various models quantum loop corrections still remains in general a challenging open question. This fact is particularly true for the models constructed by using SW map expansion in the NC parameter θ, including NCSM [9–12]. However the resulting models are very useful as effective field theories including relevant phenomenology [13–16] and their one-loop quantum properties [17–27].

Quite recently, θ-exact SW map expansions, in the framework of covariant noncommutative quantum gauge field theory [28], were applied in loop computation [29–32] and phenomenology [33, 34].

1. Phenomenological motivation

Our main motivation to discuss a constraint on the scale of the NC guge field theory (NCGFT), Λ_{NC}, arising from ultra-high energy cosmic ray experiments involving ν-nucleon inelastic cross section, is physically due to the possibility of a direct coupling of ν's to gauge bosons in the NC background [12], which plays the role of an external field in the theory (see i.e. Fig. 1). The observation of ultra-high energy (UHE) ν's from extraterrestrial sources would open a new window to look to the cosmos, as such ν's may easily escape very dense material backgrounds around local astrophysical objects, giving thereby information on regions that are otherwise hidden to any other means of exploration. In addition, ν's are not deflected on their way to the earth by various
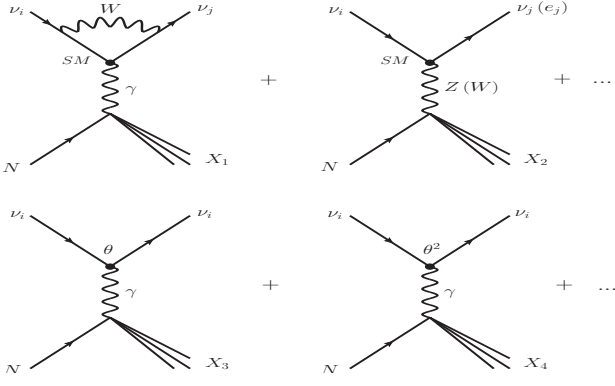
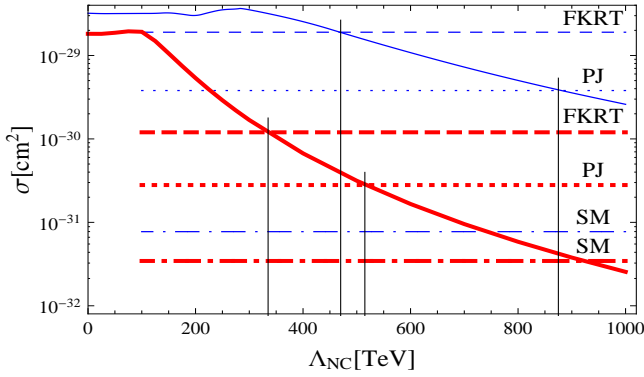FIG. 1: Diagrams contributing to $\nu N \to \nu + X$ processes.

FIG. 2: $\nu N \to \nu + anything$ cross sections vs. Λ_{NC} for $E_\nu = 10^{10}$ GeV (thick lines) and $E_\nu = 10^{11}$ GeV (thin lines). FKRT and PJ lines are the upper bounds on the ν-nucleon inelastic cross section, denoting different estimates for the cosmogenic ν-flux. SM denotes the SM total (charged current plus neutral current) ν-nucleon inelastic cross section. The vertical lines denote the intersections of our curves with the RICE results.
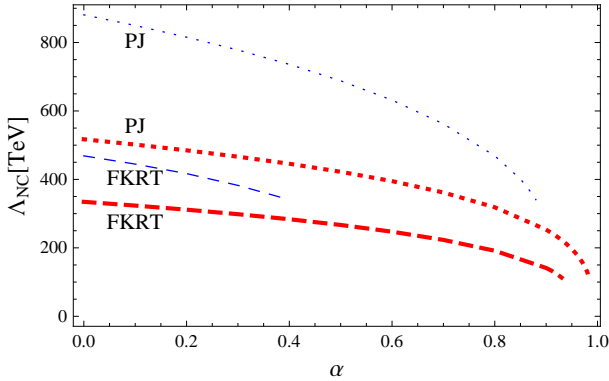

FIG. 3: The intersections of our curves with the RICE results (cf. Fig.2) as a function of the fraction of Fe nuclei in the UHE cosmic rays. The terminal point on each curve represents the highest fraction of Fe nuclei above which no useful information on Λ_{NC} can be inferred with our method.
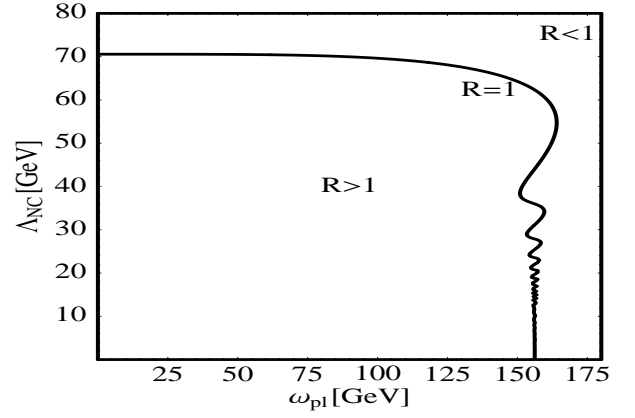

FIG. 4: The plot of scale Λ_{NC} versus the plasmon frequency ω_{pl} with $R = 1$
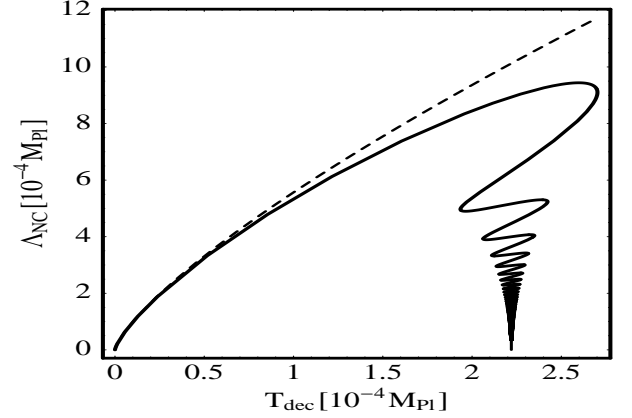

FIG. 5: The plot of the scale Λ_{NC} versus T_{dec} for perturbative/exact solution (dashed/full curve).

magnetic fields, pointing thus back to the direction of distant UHE cosmic-ray source candidates. This could also help resolving the underlying acceleration in astrophysical sources.

Since the GZK-structure in the energy spectrum of UHE cosmic rays at $\sim 4 \times 10^{19}$ eV has been observed recently with high statistical accuracy [35], the flux of the so-called cosmogenic ν's, arising from photo-pion production on the cosmic microwave background $p\gamma_{CMB} \to \Delta^* \to N\pi$ and subsequent pion decay, is now guaranteed to exist. Although estimates for the cosmogenic ν-flux are very model-dependent, primarily due to our insufficient knowledge of the nature and the origin of UHE cosmic rays, possible ranges for the size of the flux of cosmogenic ν's can be obtained from separate analysis of the data from various large-scale observatories [36, 37].

Considering the uncertainty in the flux of cosmogenic ν's regarding the chemical composition of UHE cosmic rays see detailed discussion in [33]. Employing the upper

bound on the νN cross section derived from the RICE Collaboration search results [38] at $E_\nu = 10^{11}$ GeV (4×10^{-3} mb for the FKRT ν-flux [36])), one can infer from θ-truncated model on the NC scale $\Lambda_{\rm NC}$ to be greater than 455 TeV, a really strong bound. Here we have used $\theta^{\mu\nu} \equiv c^{\mu\nu}/\Lambda_{\rm NC}^2$ such that the matrix elements of c are of order one.

One should however be careful and suspect this result as it has been obtained from the conjecture that the θ-expansion stays well-defined in the kinematical region of interest. Although a heuristic criterion for the validity of the perturbative θ-expansion, $\sqrt{s}/\Lambda_{\rm NC} \lesssim 1$, with $s = 2E_\nu M_N$, would underpin our result on $\Lambda_{\rm NC}$, a more thorough inspection on the kinematics of the process does reveal a more stronger energy dependence $E_\nu^{1/2} s^{1/4}/\Lambda_{\rm NC} \lesssim 1$. In spite of an additional phase-space suppression for small x's in the θ^2-contribution [14] of the cross section relative to the θ-contribution, we find an unacceptably large ratio $\sigma(\theta^2)/\sigma(\theta) \simeq 10^4$, at $\Lambda_{\rm NC} = 455$ TeV. Hence, the bound on $\Lambda_{\rm NC}$ obtained this way is incorrect, and our last resort is to modify the model adequately to include the full-θ resummation, thereby allowing us to compute nonperturbatively in θ.

The behavior of the cross section with the NC scale at fixed $E_\nu = 10^{10}$ GeV and $E_\nu = 10^{11}$ GeV, together with the upper bounds depending on the actual size of the cosmogenic ν-flux (FKRT [36] and PJ [37]) as well as the total SM cross sections at these energies, are depicted in our Figure 2. In order to maximize the NC θ-exact effect we choose $c_{01} - c_{13} = c_{02} - c_{23} = c_{03} = 1$.

If future data confirm that UHE cosmic rays are composed mainly of Fe nuclei, as indicated, for the time being, by the PAO data, then still valuable information on $\Lambda_{\rm NC}$ can be obtained with our method, as seen in Fig.3 . Here we see the intersections of our curves with the RICE results (cf. Fig.2) as a function of the fraction α of Fe nuclei in the UHE cosmic rays. On top of results, presented in Figs.2 and 3, we also have the NC scale given as a function of the plasmon frequency (Fig.4) and as a function of the T_{dec} from BBN (Fig.5), respectively. All results depicted in Figs.2-5, shows closed/convergent behaviour. In our opinion those were the strong signs to continue research towards quantum properties of such θ-exact noncommutative gauge field theory model.

2. Covariant θ-exact U$_\star(1)$ model

Neutrinos do not carry an electromagnetic charge and hence do not couple directly to photons. However, in the presence of spacetime NC, it is possible to couple photons to neutral and "chiral" fermion particles [33, 34].

We start with the following SW type of NC U$_\star(1)$ gauge model:

$$S = \int -\frac{1}{4} F^{\mu\nu} \star F_{\mu\nu} + i\bar{\Psi} \star \slashed{D}\Psi, \qquad (4)$$

with definitions of the nonabelian NC covariant derivative and the field strength, respectively:

$$\begin{aligned}
D_\mu \Psi &= \partial_\mu \Psi - i[A_\mu \stackrel{\star}{,} \Psi], \\
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu \stackrel{\star}{,} A_\nu].
\end{aligned} \qquad (5)$$

All the fields in this action are images under (hybrid) Seiberg-Witten maps of the corresponding commutative fields a_μ and ψ. Here we shall interpret the NC fields as valued in the enveloping algebra of the underlying gauge group. This naturally corresponds to an expansion in powers of the gauge field a_μ and hence in powers of the coupling constant e. At each order in a_μ we shall determine θ-exact expressions.

In the next step we expand the action in terms of the commutative gauge parameter λ and fields a_μ and ψ using the SW map solution [30] up to the $\mathcal{O}(a^3)$ order:

$$\begin{aligned}
A_\mu &= a_\mu - \frac{1}{2}\theta^{\nu\rho} a_\nu \star_2 (\partial_\rho a_\mu + f_{\rho\mu}), \\
\Psi &= \psi - \theta^{\mu\nu} a_\mu \star_2 \partial_\nu \psi \\
&+ \frac{1}{2}\theta^{\mu\nu}\theta^{\rho\sigma}\Big\{ (a_\rho \star_2 (\partial_\sigma a_\mu + f_{\sigma\mu})) \star_2 \partial_\nu \psi \\
&+ 2a_\mu \star_2 (\partial_\nu(a_\rho \star_2 \partial_\sigma \psi)) - a_\mu \star_2 (\partial_\rho a_\nu \star_2 \partial_\sigma \psi) \\
&- \big(a_\rho \partial_\mu \psi (\partial_\nu a_\sigma + f_{\nu\sigma}) - \partial_\rho \partial_\mu \psi a_\nu a_\sigma \big)_{\star_3} \Big\}, \\
\Lambda &= \lambda - \frac{1}{2}\theta^{ij} a_i \star_2 \partial_j \lambda, \qquad (6)
\end{aligned}$$

with Λ being the NC gauge parameter and $f_{\mu\nu}$ is the abelian commutative field strength $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$.

The generalized Mojal-Weyl star products $\star_2$ and $\star_3$, appearing in (6), are defined, respectively, as

$$\begin{aligned}
f(x) \star_2 g(x) &= [f(x) \stackrel{\star}{,} g(x)] \\
&= \frac{\sin \frac{\partial_1 \theta \partial_2}{2}}{\frac{\partial_1 \theta \partial_2}{2}} f(x_1) g(x_2) \Big|_{x_1=x_2=x}, \quad (7)
\end{aligned}$$

$$\begin{aligned}
(f(x)g(x)h(x))_{\star_3} &= \Bigg(\frac{\sin(\frac{\partial_2 \theta \partial_3}{2}) \sin(\frac{\partial_1 \theta (\partial_2 + \partial_3)}{2})}{\frac{(\partial_1 + \partial_2)\theta \partial_3}{2} \frac{\partial_1 \theta (\partial_2 + \partial_3)}{2}} \\
&+ \{1 \leftrightarrow 2\} \Bigg) f(x_1) g(x_2) h(x_3) \Big|_{x_i = x}, \\
& \qquad\qquad\qquad\qquad\qquad (8)
\end{aligned}$$

where $\star$ is associative but noncommutative, while $\star_2$ and $\star_3$ are both commutative but nonassociative.

The resulting expansion defines the one-photon-two-fermion, the two-photon-two-fermion and the three-photon vertices, θ-exactly:

$$\begin{aligned}
S_g &= \int i\partial_\mu a_\nu \star [a^\mu \stackrel{\star}{,} a^\nu] \\
&+ \frac{1}{2}\partial_\mu \left(\theta^{\rho\sigma} a_\rho \star_2 (\partial_\sigma a_\nu + f_{\sigma\nu})\right) \star f^{\mu\nu}. \qquad (9)
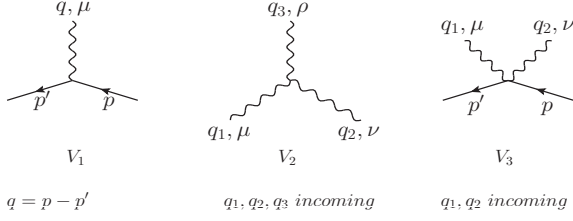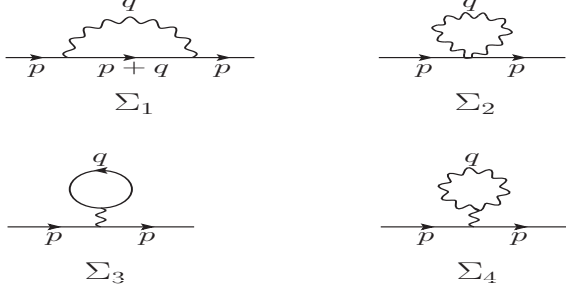\end{aligned}$$

FIG. 6: Three- and your-field vertices

FIG. 7: One-loop self-energy of a massless neutrino

The photon-fermion interaction up to 2-photon-2-fermion terms can derived by using the first order gauge field and the second order fermion field expansion,

$$
\begin{aligned}
S_f &= \int \bar{\psi}\gamma^\mu[a_\mu \overset{\star}{,} \psi] + (\theta^{ij}\partial_i\bar{\psi} \star_2 a_j)\gamma^\mu[a_\mu \overset{\star}{,} \psi] \\
&+ i(\theta^{ij}\partial_i\bar{\psi} \star_2 a_j)\slashed{\partial}\psi - i\bar{\psi} \star \slashed{\partial}(\theta^{ij}a_i \star_2 \partial_j\psi) \\
&- \bar{\psi}\gamma^\mu[a_\mu \overset{\star}{,} \theta^{ij}a_i \star_2 \partial_j\psi] \\
&- \bar{\psi}\gamma^\mu[\tfrac{1}{2}\theta^{ij}a_i \star_2 (\partial_j a_\mu + f_{j\mu}) \overset{\star}{,} \psi] \\
&- i(\theta^{ij}\partial_i\bar{\psi} \star_2 a_j)\slashed{\partial}(\theta^{kl}a_k \star_2 \partial_l\psi) \\
&+ \frac{i}{2}\theta^{ij}\theta^{kl}\big[(a_k \star_2 (\partial_l a_i + f_{li})) \star_2 \partial_j\bar{\psi} \\
&+ 2a_i \star_2 (\partial_j(a_k \star_2 \partial_l\bar{\psi})) - a_i \star_2 (\partial_k a_j \star_2 \partial_l\bar{\psi}) \\
&+ \big(a_i\partial_k\bar{\psi}(\partial_j a_l + f_{jl}) - \partial_k\partial_i\bar{\psi}a_j a_l\big)_{\star_3}\big]\slashed{\partial}\psi \\
&+ \frac{i}{2}\theta^{ij}\theta^{kl}\bar{\psi}\slashed{\partial}\big[(a_k \star_2 (\partial_l a_i + f_{li})) \star_2 \partial_j\psi \\
&+ 2a_i \star_2(\partial_j(a_k \star_2 \partial_l\psi)) - a_i \star_2 (\partial_k a_j \star_2 \partial_l\psi) \\
&+ \big(a_i\partial_k\psi(\partial_j a_l + f_{jl}) - \partial_k\partial_i\psi a_j a_l\big)_{\star_3}\big].
\end{aligned}
\tag{10}
$$

Note that actions for gauge and matter fields obtained above, (9) and (10) respectively, are nonlocal objects due to the presence of the (generalized) star products: $\star$, $\star_2$ and $\star_3$. Feynman rules from above actions, represented in Fig.6, are given explicitly in [32].

3. Neutrino two-point function

As depicted in Fig. 7, there are four Feynman diagrams contributing to the ν-self-energy at one-loop. With the aid of (10), we have verified by explicit calculation that the 4-field tadpole (Σ_2) does vanish. The 3-fields tadpoles (Σ_3 and Σ_4) can be ruled out by invoking the NC charge conjugation symmetry [11]. Thus only the Σ_1 diagram needs to be evaluated. In spacetime of the dimensionality D we obtain

$$
\begin{aligned}
\Sigma_1 &= \mu^{4-D}\int \frac{d^D q}{(2\pi)^D}\left(\frac{\sin\frac{q\theta p}{2}}{\frac{q\theta p}{2}}\right)^2 \frac{1}{q^2}\frac{1}{(p+q)^2} \\
&\quad \cdot \;\{(q\theta p)^2(4-D)(\slashed{p}+\slashed{q}) \\
&\quad + (q\theta p)\left[\slashed{q}(2p^2+2p\cdot q) - \slashed{p}(2q^2+2p\cdot q)\right] \\
&\quad + \left[\slashed{p}(\tilde{q}^2(p^2+2p\cdot q) - q^2(\tilde{p}^2+2\tilde{p}\cdot\tilde{q}))\right. \\
&\quad + \left.\slashed{q}(\tilde{p}^2(q^2+2p\cdot q) - p^2(\tilde{q}^2+2\tilde{p}\cdot\tilde{q}))\right]\},
\end{aligned}
\tag{11}
$$

where $\tilde{p}^\mu = (\theta p)^\mu = \theta^{\mu\nu}p_\nu$, and in addition $\tilde{\tilde{p}}^\mu = (\theta\theta p)^\mu = \theta^{\mu\nu}\theta_{\nu\rho}p^\rho$. To perform computations of those integrals using the dimensional regularization method, we first use the Feynman parametrization on the quadratic denominators, then the Heavy Quark Effective theory (HQET) parametrization [39] is used to combine the quadratic and linear denominators. In the next stage we use the Schwinger parametrization to turn the denominators into Gaussian integrals. Evaluating the relevant integrals for $D = 4 - \epsilon$ in the limit $\epsilon \to 0$, we obtain the closed form expression for the self-energy as

$$
\Sigma_1 = \gamma_\mu\left[p^\mu A + (\theta\theta p)^\mu \frac{p^2}{(\theta p)^2} B\right],
\tag{12}
$$

$$
\begin{aligned}
A &= \frac{-1}{(4\pi)^2}\left[p^2\left(\frac{\mathrm{tr}\theta\theta}{(\theta p)^2} + 2\frac{(\theta\theta p)^2}{(\theta p)^4}\right)A_1\right. \\
&\quad + \left.\left(1 + p^2\left(\frac{\mathrm{tr}\theta\theta}{(\theta p)^2} + \frac{(\theta\theta p)^2}{(\theta p)^4}\right)\right)A_2\right],
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
A_1 &= \frac{2}{\epsilon} + \ln(\mu^2(\theta p)^2) + \ln(\pi e^{\gamma_E}) \\
&+ \sum_{k=1}^{\infty}\frac{(p^2(\theta p)^2/4)^k}{\Gamma(2k+2)}\left(\ln\frac{p^2(\theta p)^2}{4} + 2\psi_0(2k+2)\right),
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
A_2 &= -\frac{(4\pi)^2}{2}B = -2 \\
&+ \sum_{k=0}^{\infty}\frac{(p^2(\theta p)^2/4)^{k+1}}{(2k+1)(2k+3)\Gamma(2k+2)}\left(\ln\frac{p^2(\theta p)^2}{4}\right. \\
&\quad - \left.2\psi_0(2k+2) - \frac{8(k+1)}{(2k+1)(2k+3)}\right),
\end{aligned}
\tag{15}
$$

with $\gamma_E \simeq 0.577216$ being Euler's constant.

The $1/\epsilon$ UV divergence could in principle be removed by a properly chosen counterterm. However (as already mentioned) due to the specific momentum-dependent coefficient in front of it, a nonlocal form for it is required.

It is important to stress here that amongst other terms contained in both coefficients A_1 and A_2, there are structures proportional to

$$
(p^2(\theta p)^2)^{n+1}(\ln(p^2(\theta p)^2))^m, \quad \forall n \text{ and } m = 0,1.
\tag{16}
$$

The numerical factors in front of the above structures are rapidly-decaying, thus series are always convergent for finite argument, as we demonstrate in [32].

4. UV/IR mixing

Turning to the UV/IR mixing problem, we recognize a soft UV/IR mixing term represented by a logarithm,

$$\Sigma_{\text{UV/IR}} = -\slashed{p}\, p^2 \left(\frac{\text{tr}\theta\theta}{(\theta p)^2} + 2\frac{(\theta\theta p)^2}{(\theta p)^4} \right) \cdot \frac{2}{(4\pi)^2} \ln|\mu(\theta p)|.$$
(17)

Instead of dealing with nonlocal counterterms, we take a different route here to cope with various divergences besetting (12). Since $\theta^{0i} \neq 0$ makes a NC theory nonunitary [40], we can, without loss of generality, chose θ to lie in the $(1, 2)$ plane

$$\theta^{\mu\nu} = \frac{1}{\Lambda_{\text{NC}}^2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$
(18)

Automatically, this produces

$$\frac{\text{tr}\theta\theta}{(\theta p)^2} + 2\frac{(\theta\theta p)^2}{(\theta p)^4} = 0, \; \forall p.$$
(19)

With (19), Σ_1, in terms of Euclidean momenta, receives the following form:

$$\Sigma_1 = \frac{-1}{(4\pi)^2} \gamma_\mu \left[p^\mu \left(1 + \frac{\text{tr}\theta\theta}{2} \frac{p^2}{(\theta p)^2} \right) - 2(\theta\theta p)^\mu \frac{p^2}{(\theta p)^2} \right] A_2.$$
(20)

By inspecting (15) one can be easily convinced that A_2 is free from the $1/\epsilon$ divergence and the UV/IR mixing term, being also well-behaved in the infrared, in the $\theta \to 0$ as well as $\theta p \to 0$ limit. We see, however, that the two terms in (20), one being proportional to $\slashed{p}$ and the other proportional to $\tilde{\slashed{p}}$, are still ill-behaved in the $\theta p \to 0$ limit. If, for the choice (18), P denotes the momentum in the $(1, 2)$ plane, then $\theta p = \theta P$. For instance, a particle moving inside the NC plane with momentum P along the one axis, has a spatial extension of size $|\theta P|$ along the other. For the choice (18), $\theta p \to 0$ corresponds to a zero momentum projection onto the $(1, 2)$ plane. Thus, albeit in our approach the commutative limit $(\theta \to 0)$ is smooth at the quantum level, the limit when an extended object (arising due to the fuzziness of space) shrinks to zero, is not. We could surely claim that in our approach the UV/IR mixing problem is considerably softened; on the other hand, we have witnessed how the problem strikes back in an unexpected way. This is, at the same time, the first example where this two limits are not degenerate.

5. Neutrino dispersion relations

In order to probe physical consequence of the 1-loop quantum correction, with Σ_{1-loop_M} from Eq. (3.25) in [32], we consider the modified propagator

$$\frac{1}{\tilde{\slashed{\Sigma}}} = \frac{1}{\slashed{p} - \Sigma_{1-loop_M}} = \frac{\tilde{\slashed{\Sigma}}}{\Sigma^2}.$$
(21)

We further choose the NC parameter to be (18) so that the denominator is finite and can be expressed explicitly:

$$\Sigma^2 = p^2 \left[\hat{A}_2^2 \left(\frac{p^4}{p_r^4} + 2\frac{p^2}{p_r^2} + 5 \right) - \hat{A}_2 \left(6 + 2\frac{p^2}{p_r^2} \right) + 1 \right], \quad (23)$$

where p_r represents r-component of the momentum p in a cylindrical spatial coordinate system and $\hat{A}_2 = e^2 A_2/(4\pi)^2 = -B/2$.

From above one see that $p^2 = 0$ defines one set of the dispersion relation, corresponding to the dispersion for the massless neutrino mode, however the denominator Σ^2 has one more coefficient Σ' which could also induce certain zero-points. Since the $\hat{A}_2$ is a function of a single variable $p^2 p_r^2$, with $p^2 = \text{p}_0^2 - \text{p}_1^2 - \text{p}_2^2 - \text{p}_3^2$ and $\text{p}_r^2 = \text{p}_1^2 + \text{p}_2^2$, the condition $\Sigma' = 0$ can be expressed as a simple algebraic equation

$$\hat{A}_2^2 z^2 - 2\left(A_2 - \hat{A}_2^2 \right) z + \left(1 - 6\hat{A}_2 + 5\hat{A}_2^2 \right) = 0, \quad (22)$$

of new variables $z := p^2/p_r^2$, in which the coefficients are all functions of $y := p^2 p_r^2/\Lambda_{\text{NC}}^4$.

The two formal solutions of the equation (22)

$$z = \frac{1}{\hat{A}_2} \left[\left(1 - \hat{A}_2 \right) \pm 2\left(\hat{A}_2 - \hat{A}_2^2 \right)^{\frac{1}{2}} \right], \quad (23)$$

are birefringent. The behavior of solutions (23), is next analyzed at two limits $y \to 0$, and $y \to \infty$.

The low-energy regime: $p^2 p_r^2 \ll \Lambda_{\text{NC}}^4$
For $y \ll 1$ we set $\hat{A}_2$ to its zeroth order value $e^2/8\pi^2$,

$$p^2 \sim \left(\left(\frac{8\pi^2}{e^2} - 1 \right) \pm 2\left(\frac{8\pi^2}{e^2} - 1 \right)^{\frac{1}{2}} \right) \cdot p_r^2$$
$$\simeq (859 \pm 59) \cdot p_r^2, \quad (24)$$

obtaining two (approximate) zero points. From the definition of p^2 and p_r^2 we see that both solutions are real and positive. Taking into account the higher order (in y) correction these poles will locate nearby the real axis of the complex p_0 plane thus correspond to some metastable modes with the above defined dispersion relations. As we can see, the modified dispersion relation (24) does not depend on the noncommutative scale, therefore it introduces a discontinuity in the $\Lambda_{\text{NC}} \to \infty$ limit, which is not unfamiliar in noncommutative theories.

The high-energy regime: $p^2 p_r^2 \gg \Lambda_{NC}^4$

At $y \gg 1$ we analyze the asymptotic behavior of

$$A_2 \sim \frac{i\pi^2}{8}\sqrt{y}\left(1 - \frac{16i}{\pi y}e^{-\frac{i}{2}\sqrt{y}}\right) + \mathcal{O}\left(y^{-1}\right), \qquad (25)$$

from [32], therefore (23) can be reduced to

$$z \sim -1 \pm 2i \quad \rightarrow \quad p_0^2 \sim p_3^2 \pm 2ip_r^2. \qquad (26)$$

We thus reach two unstable deformed modes besides the usual mode $p^2 = 0$ in the high energy regime. Here again the leading order deformed dispersion relation does not depend on the noncommutative scale Λ_{NC}.

6. Neutrino two-point function from alternative action

Using the Feynman rule of the alternative action (2.15) from Ref [32], we find the following contribution to the neutrino self-energy from diagram Σ_1

$$\Sigma_{1_{alt}} = \frac{\not{p}}{(4\pi)^2}\left[\frac{8}{3}\frac{1}{(\theta p)^2}\left(\frac{\mathrm{tr}\theta\theta}{(\theta p)^2} + 4\frac{(\theta\theta p)^2}{(\theta p)^4}\right)\right]. \qquad (27)$$

The detailed computation is presented in Ref. [32] (Appendix B). Using the same technique and the choice (18) we have found modified propagator of the following form

$$\frac{i}{\Sigma_{1_{alt}}} = \frac{i}{\not{p}}\frac{p_r^4}{p_r^4 + \frac{\Lambda_{NC}^4}{3\pi^2}} = \Pi\frac{i\not{p}}{p^2}. \qquad (28)$$

Since the only pole for p_0 comes from $p^2 = 0$, there is no modified dispersion relation. However, we can see that $\Pi = 0$ when $|p_r| = 0$, therefore there could be directional dependent effect on the propagation. One may try a source function $f(x_\mu)$, then (neglecting some details about time ordering) for a wave function $\phi(x)$ which propagates according to the scalar part of the modified propagator (28) one could have

$$\phi(x_\mu) \sim \int d^4x' \int d^4p \frac{i\Pi}{p^2}e^{ip(x-x')}f(x') \\ \sim \int d^4p \frac{i\Pi}{p^2}e^{ipx}\tilde{f}(p). \qquad (29)$$

The direction dependent factor Π could then take zero value for $|p_r| = 0$ modes, thus give suppressive effect.

We have to notice that there are no hard $1/\epsilon$ UV divergent and no logarithmic UV/IR mixing terms, and the finite terms like in A_1 and A_2 are also absent. Thus the subgraph Σ_1 does not require any counter-term. However, the result (27), does express powerful UV/IR mixing effect, that is in terms of scales terms, the $\Sigma_{1_{alt}}$ experience the forth-power of the *NC-scale/momentum-scale* ratios $\sim |p|^{-2}|\theta p|^{-2}$ in (27), i.e. we are dealing with the $\Sigma_{1_{alt}} \sim \not{p}\left(\Lambda_{NC}/p\right)^4$ within the ultraviolet and infrared limits for Λ_{NC} and p, respectively.

7. Discussion and conclusions

If new physics were capable to push the neutrino-nucleon inelastic cross section three orders of magnitude beyond the standard-model (SM) prediction, then ultra-high energy (UHE) neutrinos would have already been observed at neutrino observatories. We use such a constraint to reveal information on the scale of noncommutativity Λ_{NC} in noncommutative gauge field theories (NCGFT) where ν's possess a tree-level coupling to photons in a generation-independent manner. In the energy range of interest (10^7 to 10^8 TeV for example) where there is always energy of the system (E) larger than the NC scale ($E/\Lambda_{NC} > 1$), the perturbative expansion in terms of Λ_{NC} retains no longer its meaningful character, thus it is forcing us to resort to those NC field-theoretical frameworks involving the full θ-resumation. Our numerical analysis of the contribution to the process coming from the photon exchange, pins impeccably down a lower bound on Λ_{NC} to be as high as around up to $\mathcal{O}(10^3)$ TeV, depending on the cosmogenic ν-flux.

We should stress that from the same actions (9, 10), but for three different cosmological laboratories, that is from UHE cosmic ray neutrino scatterings on nuclei [33], from the BBN nucleosynthesis and from the reheating phase after inflation [41], we obtain very similar, a quite strong bounds on the NC scale, of the order of 10^3 TeV. Note in particular that all results depicted in Figs.2-5, show closed-convergent forms.

Next we discuss θ-exact computation of the one-loop quantum correction to the ν-propagator. We in particular evaluate the neutrino two-point function, and demonstrate how quantum effects in the θ-exact SW map approach to NCGFT's, together with a combination of Schwinger, Feynman, and HQET parameterization, reveal a much richer structure yielding the one-loop quantum correction in a closed form.

The closed form general expression for the neutrino self-energy (12) contains in (14) both a hard $1/\epsilon$ UV term and the celebrated UV/IR mixing term with a logarithmic infrared singularity $\ln\sqrt{(\theta p)^2}$. Results shows complete decoupling of the UV divergent term from softened UV/IR mixing term and from the finite terms as well. Our deformed dispersion relations at both the low and high energies and at the leading order do not depend on the noncommutative scale Λ_{NC}.

The low energy dispersion relation (24) is, in principle, capable of generating a direction dependent superluminal velocity, this can be seen clearly from the maximal attainable velocity of the neutrinos

$$\mathrm{v}_{max} = \frac{dE}{d|\vec{p}|} \sim \sqrt{1 + (859 \pm 59)\sin^2\vartheta}, \qquad (30)$$

where ϑ is the angle with respect to the direction perpendicular to the NC plane. This gives one more example how such spontaneous breaking of Lorentz symmetry (via the θ-background) could affect the particle

kinematics through quantum corrections (even without divergent behavior like UV/IR mixing). On the other hand one can also see that the magnitude of super-luminosity is in general very large in our model, thus seems contradicting various observations. The currently largest superluminal velocity for neutrinos, $(v - c)/c = (2.37 \pm 0.32(\text{stat.})^{+0.34}_{-0.24}(\text{sys.})) \times 10^{-5}$ reported by the OPERA collaboration [5], has been found to be suffering from multiple technical difficulties lately [42]. Other experiments suggests much smaller values [43–45]. The authors consider here, on the other hand, that the large superluminal velocity issue may be reduced/removed by taking into account several further considerations:

(a) First, the reason to select a constant nonzero θ background in this paper is computational simplicity. The results will, however, still hold for a NC background that is varying sufficiently slowly with respect to the scale of noncommutativity. There is no physics reason to expect θ to be a globally constant background *ether*. In fact, if the θ background is only nonzero in tiny regions (NC bubbles) the effects of the modified dispersion relation will be suppressed macroscopically. Certainly a better understanding of possible sources of NC is needed.

(b) Second, in our computation we considered only the purely noncommutative neutrino-photon coupling, it has been pointed out that modified neutrino dispersion relation could open decay channels within the commutative standard model framework [46]. In our case this would further provide decay channel(s) which can bring super-luminal neutrinos to normal ones.

(c) Third, as we have stated in the introduction, model 1 is not the only allowed deformed model with noncommutative neutrino-photon coupling. And as we have shown for our model 2, there could be no modified dispersion relation(s) for deformation(s) other than 1, therefore it is reasonable to conjecture that Seiberg-Witten map freedom may also serve as one possible remedy to this issue.

The essential difference of our results as compared to [47] is that in our case both terms are proportional to the spacetime noncommutativity dependent θ-ratio factor in (19), which arise from the natural non-locality of our actions and does not depend on the noncommutative scale, but only on the scale-independent structure of the non-commutative θ-ratios. Besides the divergent terms, a new spinor structure $(\theta\theta p)$ with finite coefficients emerges in our computation, see (12)-(15). All these structures are proportional to p^2, therefore if appropriate renormalization conditions are imposed, the commutative dispersion relation $p^2 = 0$ can still hold, as a part of the full set of solutions obtained in (23).

Finally, we mention that our approach to UV/IR mixing should not be confused with the one based on a theory with UV completion ($\Lambda_{\text{UV}} < \infty$), where a theory becomes an effective QFT, and the UV/IR mixing manifests itself via a specific relationship between the UV and the IR cutoffs [48, 49].

The properties summarized above are previously unknown features of θ-exact NC gauge field theory. They appear in the model with the action presented in section 2. The alternative action, and the corresponding ν-self-energy (27), has less striking features, but it does have it's own advantages due to the absence of a hard UV divergences, and the absence of complicated finite terms. The structure in (27) is different (it is *NC-scale/energy* dependent) with respect to the NC scale-independent structure from (19), as well as to the structure arising from fermion self-energy computation in the case of ⋆-product only unexpanded theories [47, 50]. However, (27) does posses powerful UV/IR mixing effect. This is fortunate with regard to the use of low-energy NCQFT as an important window to quantum gravity [51] and holography [52].

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